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REDUCTION OF LAWS AND CONCEPTS*

1. Introduction

Scientific practice is almost by definition a matter of reduction, either in the sense of transforming something into something else, or in the sense of concentrating on some aspects, provisionally neglecting some other aspects, to be accounted for later. The second sense of reduction is the "reculer pour mieux sauter" – sense, so aptly called by Leszek Nowak [1980]: idealization and (subsequent) concretization. Nowak and others have made it clear that this type of strategic reduction is very important in the natural as well as in the social sciences and that it is possible and useful to give an abstract characterization of this procedure when specific statements are concerned, in particular law-statements.

The first sense of reduction, let me call it the transformation sense, has received more attention in philosophy of science during the last forty years. In this context one may think primarily of the reduction of phenomena or primarily of the reduction of relations between them. Or, formulated in terms of the conceptualization of phenomena and their relations, the reduction of concepts or the reduction of laws may be the primary subject of interest. In the philosophy of science literature since 1960, the emphasis lies on reduction of laws, notwithstanding the fact that something like concept reduction frequently appears as a necessary intermediate link.

In this paper I like to report the results of separate investigations of reduction of laws (Section 2) and reduction of concepts (Section 4). In both cases the notion of (general) identities, as opposed to causal correlations, will play an important role. This distinction will be explained, and defended as a working hypothesis, in Section 3. The equally plausible as confusing talk about levels in both sorts of reduction will be explained in Section 5. I conclude with some general considerations in Section 6.

At several places there will be interesting opportunities to refer, in passing, to reduction in the strategic sense, as a matter of course. Moreover, the result of idealization and subsequent concretization with respect to law-statements will get a precise place in our five steps model of explanation of a law by a theory.

A common characteristic of the proposed analyses of reduction of laws and concepts is that the main results are decomposed into a number of standard steps. These models are typical results of the quasi-empirical study of cognitive aspects of the empirical sciences, or briefly, of *cognitive studies of science*, a type of research that may be distinguished on the one hand from *abstract philosophy of science* and on the other hand from *social studies of science*. Of course, Nowak's decomposition model of idealization and concretization is another example.

In a recent (Groningen-biased) survey of cognitive studies of science [Kuipers, 1989] I have given a short list of the possible use values of its results. Because the models to be presented are typical examples of cognitive studies of science I like to conclude this introduction with this list of possible use values:

(a) providing the "null hypothesis of the ideal course of events", which can play a guiding role in social studies of science,
(b) clarification or even solution of problems belonging to abstract philosophy of science,
(c) improvement of advanced textbooks, leading to better understanding and remembrance,
(d) playing a heuristic role in ongoing and new scientific research,
(e) playing a heuristic role in local research policy and global science policy.

2. *Reduction of laws*

In this section a decomposition model of successful reductions of laws will be presented, based on some case-studies in physics and sociology [Kuipers, 1982, 1984, 1985]. At the basis of this model lies a *five steps model* for explanation of a law by a theory (Subsection 2.2.). This model consists of five standard steps: application, aggregation, identification, correlation and approximation. According to my *pluralistic* *diagnosis* of reduction (Subsection 2.3.) there can be distinguished in the literature three different types of reduction of laws, namely when one of the steps aggregation, identification and approximation occurs in the explanation of that law. In my opinion the three different meanings discovered can explain away (use value (b)) a lot of the confusion in the "philosophical" discussion about the nature of reduction, which is heavily based on the assumption that there is only one basic type of reduction to be explicated. Moreover, the analysis enables precise definitions of distinctions occurring in the literature: homogeneous versus heterogeneous reduction, deductive versus approximative reduction, and micro-reduction versus other types of reduction.

In Subsection 2.4. I will present twelve examples of explanation of a law by a theory in terms of the decomposition model. These examples do not only illustrate the diagnosis, but also the fact that combinations of different types of reduction can occur, a fact which presumably forms one of the main causes of the controversies about the nature of reduction.

2.1. The paradigmatic example

Since Nagel [1961] presented his general analysis of reduction starting from the reduction of the ideal gas law to the kinetic theory of gases this example has become the paradigm of reduction. However, it turns out to be possible to give a more detailed analysis than Nagel did and I will start with doing so.

The (naive version of the) reduction of the ideal gas law (IGL) can be decomposed into three steps. The kinetic theory of gases (KTG) says that an isolated amount of gas consists of molecules which move and collide in accordance with Newton's laws. In the *first* step, *the application step*, these laws are applied to one molecule, colliding with the wall. In this step the *auxiliary hypothesis* (H1) is used that the collision is elastic. The result is the 'individual law' telling that the momentum exchange q equals $2mv$ (m: mass; v: velocity in the wall direction):

$$L_1 \quad q = 2mv$$

The *second* step, *the aggregation step*, consists of an ingenious aggregation, using some *statistical auxiliary hypotheses* (H2), of the momentum exchange of Avogadro's standard number of molecules (N), leading to the 'aggregated law' telling that the resulting kinetic pressure p on the wall, times the volume V of the vessel, is equal to (2/3) $N\bar{u}$:

$$\mathrm{L}_2 \quad pV = (2/3)N\bar{u},$$

where $\bar{u}$ indicates the mean kinetic energy.

In the third and last step, *the transformation* or, more specifically, the *identification step*, one introduces the *auxiliary identity hypotheses* (H3):

$$p = P \qquad \bar{u} = (3/2)(R/N)T$$

where P indicates the macroscopic pressure, T the empirical absolute temperature, and R the ideal gas constant. They enable us to derive deductively from L_2 the ideal gas law:

$$\mathrm{IGL} \quad PV = RT$$

The following scheme summarizes the argument:

$$
\begin{array}{ll}
\text{(1) application} & \underline{\text{KTG} \qquad\qquad \text{H1}} \\
\text{(2) aggregation} & \qquad\quad \underline{\text{L1} \qquad\qquad \text{H2}} \\
\text{(3) identification} & \qquad\qquad\qquad \underline{\text{L2} \qquad\qquad \text{H3}} \\
 & \qquad\qquad\qquad\qquad\quad \text{IGL}
\end{array}
$$

As a matter of fact Nagel [1961] payed attention only to the third step. I will come back to this example in Section 2.4. (Example d). In that section I will also show that the structure of the explanation of Olson's hypothesis about collective goods by utility theory (Ex. i) is equal to that of the paradigmatic example, apart from the nature of the third step.

In the next, general subsection two new steps will be introduced, *viz.* correlation and approximation, besides the three ones occurring in the example of the present subsection.

2.2. Explanation of laws: a decomposition model

By reducing a law to a theory it is explained by that theory. As a self-contained result of the research for this paper, I present as *the first gen-*

eral thesis of this section that any explanation of a law by a theory can be decomposed into five standard steps specified in the following *five steps model.*

> "Theory T explains law L" if
> *formal condition*: there are auxiliary mutually consistent hypotheses $A1$, ..., $A5$ such that L can be derived, strictly or approximately, from T using one or more of the following five steps
>
> $$\begin{array}{llllll}
\text{(1) application} & \underline{T \qquad A1} & & & & \\
\text{(2) aggregation} & & \underline{L1 \qquad A2} & & & \\
\text{(3) identification} & & & \underline{L2 \qquad A3} & & \\
\text{(4) correlation} & & & & \underline{L3 \qquad A4} & \\
\text{(5) approximation} & & & & & \dfrac{L4 \qquad A5}{L5 = L}
\end{array}$$
>
> *empirical condition*: there are good reasons to accept theory T and the required auxiliary hypotheses $A1$, ..., $A5$ as approximately true.

The empirical condition will be assumed to be satisfied throughout. This condition hides a world of problems, in particular problems of circularity and idealization. As far as problems of circularity are concerned I restrict myself to the remark that the explanatory theory usually has proven its merits already in other situations, whereas the required auxiliary hypotheses frequently derive their credibility from the explanation itself.

All five (types of) auxiliary hypotheses can be assumed to be idealizations, asking for further concretization, leading to the concretization of the explanation. Moreover, the approximation step (5) includes idealization in still another way, such that, in contrast to the other steps, it is a non-deductive step. If it occurs, I will speak of an *approximative* explanation, if it does not occur it is a *deductive* explanation. Here I have taken "deductive" in the straightforward standard sense, not excluding the possibility that an approximative step can be reconstrued as a non-standard deductive step as described by Pearce and

Rantala [1985] or as a kind of deductive counterfactual reasoning along the lines of Gärdenfors [1988].

In the following stepwise elucidation I will do as if all five steps always occur precisely one time and in the indicated order.

Ad(1) application: T is tailored to the kind of object, system or situation where L is, implicitly or explicitly, about. This modelling usually requires a lot of auxiliary hypotheses, in particular specification hypotheses concerning variables of T. Together with T, this leads to some law, which may in the face of an aggregation step be called an 'individual law'.

Ad(2) aggregation: the total effect of the 'individual law' for many objects is calculated by a suitable addition, or synthesis if more than one type of individual law is involved. The required auxiliary hypotheses are of course primarily statistical hypotheses. The resulting law is called an 'aggregated law'.

Ad(3) identification: the aggregated law is transformed with the aid of one or more identity hypotheses, ontologically identifying object and predicate terms of L with terms (definable in terms) of T. The resulting law need not yet be completely stated in terms of L.

Ad(4) correlation: the resulting law is now transformed with the aid of some causal hypotheses, correlating terms of L with (defined) terms of T. In Sect. 3 I will discuss extensively the distinction between identities and correlations.

Ad(5) approximation: the deductively derived law is now simplified by some mathematical approximation device, in particular counterfactual idealization, justified by an auxiliary hypothesis stating implicitly or explicitly, for instance, that some relevant term is relatively small. In this way, the last step provides a last opportunity to get rid of terms not occurring in L, *viz.* terms disappearing when they assume some extreme value.

The approximation step, of which an example will be sketched in the next subsection (Ex. a), is more or less the opposite of concretization in the sense of Nowak [1980]. Hence, from this perspective, step (5) might better be called *deconcretization*. This would moreover avoid the misunderstanding that the approximation step leads to a closer approximation of the truth, for as a rule, it will bring us further from the truth. However, from a mathematical point of view, '(mathematical) approximation' seems the most general term to cover all intended cases. It is, for instance, still unclear to me whether it can be defended to include in a model of explanation an idealization step which is not at the same time an approximation in the sense of Rott [1989], whereas it should include an approximation step even if it is not an idealization in Rott's sense.

So far for the stepwise elucidation. The indicated order of steps seems to be the rule, but deviations are not excluded. The only thing that seems to be excluded is that aggregation comes before application, but they may occur in some integrated way. Of course, steps may occur more than once in a complex explanation. My main claim with respect to order is that reconstruction of any particular case of a generally accepted explanation leads to a definite order.

Up to now, I did not find examples of explanation that did not fit well in the model based on the described five different kinds of steps. However, if the model might turn out to be too narrow to cope with other examples, an extended model will remain useful, provided that the number of kinds of steps remains small.

A few general remarks are in order. Steps (3) and (4) are more generally called *transformation steps*, and the auxiliary hypotheses (identities and correlations, respectively) *transformation rules*. Following Nagel [1961] and Hempel [1966] these transformation rules may also be calls *rules of correspondence* or *bridge principles*, as opposed to *internal principles*, which are seen as the constituting principles of the explanatory theory.

The following generalization seems plausible. "Theory T explains theory T^*" if and only if all laws constituting T^* can be explained by T *and mutually consistent auxiliary hypotheses*.

In the present context, the term 'law' need not be restricted to its strict sense. The five-steps-model can also be used for so-called *quasi-laws* i.e. laws which are only roughly or globally true. Moreover, *statistical laws* are of course also included. Note that explanation of a statistical law is not probabilistic or something like that, but deductive or approximative. Of course, the explanatory theory itself needs to be probabilistic in this case.

Finally, in an explanation two types of 'jumps of language' may occur. Application and approximation often imply a *jump to a sub-language*: the language of the resulting law concerns a part, mostly the relatively observational part, of the language of the preceding stage. On the other hand, transformation concerns a *heterogeneous jump of language*: identification and correlation introduce new terms, replacing terms occurring in the preceding stage.

2.3. *Terminological diagnosis of reduction*

The second general thesis of this section is the *pluralistic terminological diagnosis* that in the literature (as represented by the attached list) the concept of reduction of a law has three essentially different meanings.

> "Theory T reduces law L" or "L can be reduced to/by T" if and only if
> *systematic condition*: T explains L according to the five steps model, with at least one of the steps aggregation (2) (provided it is non-trivial), identification (3) or approximation (5),
> *temporal condition*: L has been established earlier than T.

I will not pay further attention to the temporal condition since its structural importance is of course limited.

The following specific terminology is also largely based on the literature. Reduction with an aggregation step is called reduction by aggregation or *micro-reduction*, otherwise it may be called *iso-reduction*. Reduction with a transformation step (identification or correlation) is called *heterogeneous* reduction, otherwise *homogeneous* reduction. Heterogeneous reduction based on identification may also be called reduction by identification or *identificatory* reduction. The step itself is related to what is usually meant by *concept reduction*, for essentially concepts are identified in the identification step. In Section 6 I will specify the precise relation between the identification step and the relevant type of concept reduction. Reduction with a correlation step, but without identification (hence, with aggregation and/or approximation), any be called *correlative* reduction. Note that the term heterogeneous reduction includes two quite different types of reduction (identificatory and correlative), which easily leads to confusion. Finally, reduction based on an approximation step is called reduction by approximation or *approximative* reduction, otherwise *deductive* reduction.

In sum, according to our diagnosis any example of reduction can be characterized as approximative or deductive, as heterogeneous (in particular, identificatory or correlative) or homogeneous and as micro- or iso-reduction, where at least one of the qualifications approximative, identificatory and micro- is required.

I have no counter-evidence to the claim that this diagnosis may be generalized to reduction of theories in the following, liberal way:

"theory T reduces theory T^*" if and only if T explains T^* (see the preceding subsection) such that at least one law constituting T^* is reduced by T in at least one the three distinguished senses.

The two famous formal conditions of Ernest Nagel [1961] for 'heterogeneous reduction' of a law by a theory can now be made precise: there must be transformation rules (A3 and/or A4) (*condition of connectability*) and other auxiliary hypotheses (A1 and/or A2) such that the law can be derived from the theory and all auxiliary hypotheses (*condition of derivability*). Nagel was mainly thinking of examples of which later authors [Sklar, 1967; Schaffner, 1967; Causey, 1977, among others] agreed that the transformation rules are identities. With this further restriction of the condition of connectability the two conditions specify precisely what is needed, according to the above exposition, for deductive, identificatory reduction, which may or may not be at the same time a case of micro-reduction. Note that the condition of derivability is not specifically related to reduction, for it is just the condition for deductive explanation. Unfortunately, Nagel was not very clear about formal conditions for homogeneous reduction, but his examples (a and c below) fit well in our model.

It was Nickles [1973] who very convincingly pointed out the distinction, in our terms, between reduction by aggregation and/or identification on the one hand and reduction by approximation on the other. The latter he called *domain-preserving* reduction. What he called *domain-combining* reduction is, in our term, reduction by identification (with or without aggregation) or reduction by aggregation followed by a correlation step.

From Nickles's perspective one may say that our main additional claim is that there are essentially two different kinds of non-approximative, hence deductive, reduction: by aggregation and by identification, respectively. However, I do not know of an example of explanation including identification, but neither aggregation nor approximation. Hence, it may be questioned whether there really are pure cases of deductive, identificatory (hence, heterogeneous) iso-reduction. In other words, is aggregation no prerequisite for deductive reduction or, put in still one other way, is micro-reduction not the only type of deductive reduction? If the answer is yes, then Nagel (and others) may be said to have missed the point, as far as explication is concerned, with his emphasis on connectability (transformation). I leave it as a challenge to find an example of the indicated pure type, which is usually called reduction.

The law explained or reduced by a theory may, but need not be an *empirical law*, in the theory relative sense of this term, which moreover may or may not be *independent* of the explanatory theory: the terms of the law, which will be laden with 'underlying laws and theories', are however not laden with the law itself (which makes it an empirical law), nor with the explanatory theory (which makes it independent of this theory) [Kuipers, & Zandvoort, 1985].

This inables me to conclude this section with some plausible explications of *instrumentalistic*, or partial, interpretations of explanation and reduction of theories [cf. in particular, Kemeny, & Oppenheim, 1956]. "T explains T^* instrumentalistically" if and only if T can explain all empirical laws independent from, but explained by T^*, using a consistent set of auxiliary hypotheses. "T reduces T^* instrumentalistically" if and only if T explains T^* instrumentalistically such that at least one empirical law independent from but explained by T^* is reduced by T. It will be clear that these instrumentalistic variants of explanation and reduction of theories are weaker than their corresponding 'full' versions.

2.4. Examples of reductive explanation

In this subsection I will sketch twelve examples of explanation of laws in terms of the steps that need to be made. In accordance with the diagnosis, all examples except the last one are regularly called cases of reductive explanation or simply reduction. At several occasions attention will be payed to not generally known interesting aspects. Moreover, in order to emphasize the formentioned analogy with the paradigmatic example from physics, the sociological example (i) is treated in some detail.

a. In the explanation of Galileo's law of free fall *application* of Newton's theory of gravitation to an earth-object-system, assuming that the earth's gravitation force is the only force operating on the object, is immediately followed by *approximation*, assuming that the height of fall is negligible compared with the earth's radius. Hence, only steps (1) and (5) occur, which makes it a representative example of *approximative, homogeneous iso-reduction*.

The indicated example provides a very good possibility to illustrate the approximation step. In the application step the law $L1$ (= $L4$) is derived that the acceleration $a(p)$ of a freely falling object p is proportional to $M/(R + h(t))^2$, where M and R indicate the mass and the radius of the earth, respectively, while $h(t)$ indicates the height of p at time t. Let $A5$ state the realistic hypothesis that $h(t)$ is positive but always very small compared with R, i.e. $0 < h(t) << R$. Then the law $L5$, obtained by the approximation step, is supposed to state that $a(p)$ is proportional to M/R^2. Hence, $L5$ results from $L4$ (= $L1$) by substituting in $L4$ the counterfactual idealization that $h(t) = 0$. According to $A5$ this assumption is not just counterfactual, but also justified "as a first approximation". Note, however, that, although $A5$ justifies the step from $L4$ to $L5$, $L4$ and $A5$ are together incompatible with $L5$, which implies that the step is not deductive in the standard sense.

b. The restricted validity of Newton mechanics can be explained by *approximation*, assuming low velocities, starting from an application of Einstein's special theory of relativity to the relevant system of objects. As already noted by Nickles [1973], physicists use to say that the special theory reduces to classical mechanics, whereas philosophers use to say just the opposite. The latter is reflected in our terminological diagnosis. It is perhaps generally the case that scientists say that the concretized law or theory can be reduced, by (simplifying) approximation, to its idealized predecessor. Be this as it may, the present example is again based on steps (1) and (5) and hence also illustrates *approximative, homogeneous iso-reduction*. However, the qualification homogeneous is implicitly called into question by Feyerabend [1962].

c. The reduction of rigid body mechanics to classical particle mechanics is the paradigmatic example of *deductive, homogeneous micro-reduction* ((1), (2)): the laws of motion of a rigid body are obtained by *aggregation* of the result of *application* of the laws of motion to the constituting particles, of which the mutual distances are assumed, by definition of a rigid body, to be constant [Adams, 1959].

d. The reduction of the ideal gas law to the kinetic theory of gases, sketched in Sect. 2.1., is not only the paradigm of (deductive) reduction in general, but also more specifically of *deductive, heterogeneous micro-reduction*: application (1), aggregation (2) and transformation, in particular identification (3). That and why the transformation is a case of identification I have spelled out elsewhere [Kuipers, 1982]. Whereas the transformation rule concerning pressure is easily accepted as an example of identification, this is not the case with the transformation rule concerning temperature ($u = (3/2)(R/N)T$, see Sect. 2.1.) However, on closer inspection it turns out that the relevant transformation-rule can essentially be restricted to the identification of the relation of being in the same thermal state (the basic concept for the notion of empirical temperature) with having the same mean kinetic energy.

e. In the example of the ideal gas law two of the three steps occur that are each sufficient to talk about reduction. With a detour via the law of Van der Waals we get an interesting example in which all three steps occur, hence leading to the qualification *approximative, identificatory micro-reduction*: with the aid of integrated *application* (1) and *aggregation* (2) of the kinetic theory one first derives the kinetic version of the law of Van der Waals; next, using the same *identifications* (3) as in the ideal gas case, one obtains the standard micro-version of this law. Hence, at this point we have obtained a deductive, identificatory micro-reduction of the law of Van der Waals. Finally, by *approximation* (5) (neglecting the so-called attraction and volume constant) one obtains the ideal gas law.

The present example is particularly interesting from the 'idealization and concretization' perspective. In the first place, (the macro-version of) the law of Van der Waals is the paradigm of concretization of a law, i. c. the ideal gas law. In the second place, and this is my main point in Kuipers [1985], the indicated deductive reduction of the law of Van der Waals by the kinetic theory is a concretization of the deductive reduction of the ideal gas law in the following precise general sense. The explanation of a concretization of an idealized law is said to be a concretization of the explanation of the idealized law if the former can be obtained from the latter by concretization of one or more of the principles of the explanatory theory or of the auxiliary hypotheses. In the paper mentioned I have argued that the (standard) naive and textbook accounts of the explanation of Van der Waals's law do not satisfy this condition, even in such a way that they are principally ununderstandable. The paper contains also a sketch of a sophisticated account which satisfies the concretization condition.

Of course, if concretization is really an improvement, as in the present case, an adequate explication of the notion of truthlikeness should count it as a step in the direction of the truth. Unfortunately it would take now too much space and time to elaborate my conviction that and how the explication of truthlikeness presented in Kuipers [1987] satisfies this plausible condition of adequacy.

Note finally that we have met now two versions of "the" reduction of the ideal gas law to the kinetic theory, a naive, deductive version (d) and a more sophisticated, approximate version (e). This illustrates the general fact that speaking of "the" reduction of a law to a theory is always relative to a particular version including a particular set of auxiliary hypotheses.

f. Hettema and Kuipers [1988] have shown, among other things, that the explanation of the periodic law (table) of Mendeleev by quantum atomic theory uses precisely all five steps of our model. Hence this is an example of approximative heterogeneous micro-reduction. It is particularly interesting because it shows that transformation by identification (here, the atomic number is equal to the number of electrons or protons) and by correlation (here, chemically similar behaviour is caused by analogical atomic structure) can go tegether. An example in which they occur after each other, i.e. with an intermediate, disappearing term, would be no surprise.

g. Looijen [1987] shows that the explanation of the physiological Bohr- and Halane-effects concerning the uptake and release of oxygen by the blood is a case of deductive, heterogeneous micro-reduction of these effects to the theory of chemical bonding, via the theory of allostery. The following steps turn out to occur: application, aggregation, correlation, application, simple aggregation and identification, which make it a nice illustration of an explanation where types of steps occur more than once. The specific goal of Looijen is to show that a sophisticated explication of the idea of emergent properties does not exclude micro-reduction of the lawlike occurrence of such properties.

h. The explanation of the two constitutive laws of Mendelian genetics with the aid of the principles of molecular genetics is in any case an example of *approximative, identificatory reduction*, because, after application (1), there is certainly *identification* (3) (roughly speaking, genes are identified with pieces of DNA-molecule) and approximation (5). In particular the fact that, after identification a *corrected*, i.e. concretized, version of Mendelian genetics results, which has still to be adapted (approximated) to obtain the classical laws, has drawn much attention in the philosophy of biology. Whether something like aggregation occurs, and hence whether it is a case of *micro*-reduction, has still to be investigated.

i. The next, sociological example will be elaborated in some detail, drawing upon Kuipers [1984], because it is, in my opinion, a paradigm of explanation in the social sciences. It will turn out to have in many respects the same structure as the reduction of the ideal gas law (Ex. d, see also Sect. 2.1.). It concerns the explanation by utility theory of the law or, more cautiously, the hypothesis of Olson, telling that the chance (or degree) of realization of a collective good decreases when the size of the relevant group increases.

As is well-known, utility theory claims that individuals choose that action, out of a set of alternative actions, of which they expect highest utility. In the first step, this *principle of utility maximization* is applied (1) to the choice between participation or non-participation in the realization of a collective good, at variable group size. For this step a number of auxiliary (specification) hypotheses concerning individual probability and utility assignments of the individuals are required. The result is the *individual regularity*:

L_1 each individual has its own *switch group size*, i.e. the size where he switches from participation to non-participation.

Assuming that the groups in question are statistically comparable as far as mean and variance of this individual switch group size is concerned, we obtain by aggregation (2) of $L1$ the *aggregated law*:

L_2 the larger the group the smaller the degree of participation.

Using the transformation or, more specifically, correlation hypothesis

H_c the smaller the degree of participation the smaller the chance of realization of the collective good,

we obtain by *correlation* (4) the hypothesis of Olson:

HO the larger the group the smaller the chance of realization.

Note first that the only structural difference between this example and the reduction of the ideal gas law is that the transformation in the latter case concerns identification whereas in Olson's case it is correlation.

However, due to the aggregation step, extrapolation of the terminology primarily used by philosophers of science for the natural sciences leads, according to our diagnosis, to the conclusion that there is nevertheless enough reason to call this example of explanation a case of reduction, in particular, a case of *deductive, correlative micro-reduction*.

In general, I like to call reduction of laws by utility theory *utilistic reduction*, as a kind of counterpart in sociology and economics to kinetic reduction in physics and chemistry.

Of course, the present example of utilistic reduction is based on highly idealizing auxiliary hypotheses, in particular the specification hypotheses. Hence, it should be seen as an 'ideal-typical' point of departure for sophistication, i.e. concretization, of these hypotheses and the hypothesis to be explained. The resulting explanatory argument hopefully is a concretization of the given one, leading to a deductive explanation of the concretized hypothesis, after which Olson's hypothesis itself can be re-obtained by approximation, making the example completely analogous to the concretization of the reduction of the ideal gas law (Ex. d) to that of the law of Van der Waals (Ex. e).

j. The next example illustrates at the same time three of the five use values mentioned in Section 1, *viz*. (a), (d) and (e): explaining a controversy as a deviation from the ideal, suggesting concrete new research and suggesting, more generally, a new research policy for the relevant research field. In economics there is a debate about the proper micro-foundation of macro-economics. Janssen [1987] distinguishes two approaches of this goal: the aggregation and the representativety approach, and argues in terms of the here presented five steps model of explanation why these approaches are so unsatisfactory. For the representativety approach this is easy to summarize: one tries to put the cart before the horse, namely to aggregate first and then to apply utility theory to the so-called representative subject. Janssen also shows that and how *deductive homogeneous micro-reduction* of macro-economics is possible in a straightforward way by presenting the utilistic reduction of a (probabilistically) concretized version of the standard macro-consumption function, i.e. a linear relation between national income and national consumption demand. This utilistic reduction uses application (1) and aggregation (2) and resembles in many respects the previous case of Olson. Of course, the standard version of the consumption function can be obtained by approximation (5) after the aggregation step. The resulting explanation may hence be called an *approximative* (homogeneous micro-reduction of the standard macro-consumption function.

k. Janssen, & Kuipers [1989] show that the claim of (special) collective demand theory, *viz*. that goods will be exchanged at equilibrium prices in equilibrium amounts, can be derived from the claim of (special) individual demand theory. The steps *application* (1) and *aggregation* (2) are needed for this purpose. Hence, it is again an example of *deductive homogeneous micro-reduction* in economics.

l. As promised, I conclude with an example of an explanation of a law which is, as far as I know, never called an example of reduction; according to our diagnosis this is because only the steps *application* (1) and *correlation* (4) occur. As is well-known, it is possible to explain with Mendelian genetics interbreeding laws, such as: under certain well-defined starting conditions there must result about 75% green and 25% white peas. The relevant transformation rules (such as: these three, of the four possible, combinations of alleles give rise to the feature green, the fourth combination to white) are typical examples of correlation: whole chains of causal processes are involved. Hence, the present example is a case of *deductive*, *non-reductive explanation* of a, *nota bene*, statistical law with the aid of a typically probabilistic theory.

3. Correlations and identities

In his prominent exposition of heterogeneous reduction Nagel [1961] assumed that the required rules of transformation are empirical laws or rules of correspondence. Soon after its publication it was argued [Schaffner, 1967; Sklar, 1967; Girill, 1976; Pluhar, 1978] that these rules of transformation should not be merely correlations, but identities, and hence that there is something like concept reduction involved in heterogeneous reduction. This discussion forms the background for the general decomposition model for explanation of laws in the preceding section: a correlation step, based on some (causal) correlation hypothesis and an identification step, based on an identity hypothesis. In line with this, the occurrence of an identification step in the explanation of a law was claimed to be a sufficient reason in the literature to speak about reduction of that law. The important question is, however, what precisely is the difference between correlations and identities and are there usable criteria to distinguish them.

The paradigmatic example of a synthetic or informative identity is the discovery: "The morning-star is the evening-star". What first were considered two names for two different objects, turned out to be two different names for the same object, namely, the planet Venus. This example has already occasioned very much brain-racking of philosophers, but it concerns only an individual identity, whereas in the case of reduction general identities are involved.

The most far-reaching claims seem to have been formulated by Robert Causey [1972]. According to him general identities can be represented as biconditionals satisfying the surface feature that, just like correlations, they need empirical support, but, in contrast to correlations, they are not subject to causal explanation: the quest for a causal explanation of e.g. "morning-star = evening-star" or "water = H_20" is absurd, because there is no mechanism in between. Although this surface feature is manageable in practice, it is essentially circular, because not being subject to causal explanation seems in the end only justifiable by claiming that the relevant proposition states an identity.

Among other reasons, Causey proposes in the in the light of this circularity a more profound criterion: the substitution criterion. In fact it concerns an extrapolation of a criterion that is more or less generally accepted for individual identities: if *a* is identical to *b* then it is allowed to substitute *b* for *a*, without changing the truth value, in all statements in which no personal attitude expression occurs. This depth criterion is not circular, but, in contrast to the surface criterion, it is not manageable in practice, because it can not be checked for every statement whether substitution is allowed or not.

The main difference between Causey and the more cautious advocates of identities as rules of transformation consists of the fact that Causey does not hesitate to leave not only room for so-called *thing-identities*, but also for *attribute-identities*. Thing-identities are general identities in which types of objects, belonging at first sight to different domains, are identified: "light consists of electromagnetic waves", "water consists of H_20-molecules", "genes consist of pieces of DNA-molecules", etc. Attribute-identities are general identities in which properties, relations or functions are identified. In the reduction of the ideal gas law (Section 2.4.) two nice examples occur. In the first place the usually hidden identification "the macroscopic pressure is the same as the kinetic pressure". In the second place, the identification of temperature and mean kinetic energy. In Section 2.4. (Ex. d) we already noted that the latter identification of two functions can be based on the identification of two relations: "equal thermal state is the same as equal mean kinetic energy".

It is easy to imagine different sorts of criticism on Causey's account. One point of criticism, however, can easily be put aside in the light of our diagnosis that there are three different reasons to speak of reduction of laws. As soon as a critic provides examples of reduction in which the transformation rule is undeniably a correlation (in which case it is often not even of biconditional form), then it is only necessary to show that at least one of the other two crucial steps, aggregation or approximation, occur in the decomposition of the explanation. Put in a different way, in the light of our diagnosis, the general claim of Causey should be sharpened as follows: if there occurs a transformation step in the reduction of a law, but no aggregation or approximation step, then the transformation rules should be identities.

More important than the above criticism on Causey's unconditional claim are the uttered and conceivable objections against the distinction between identities and correlations itself, against the proposed criteria for it, against the distinction between thing- and attribute-predicates, and finally, if one accepts this distinction and is prepared to admit thing-identities, against the idea of attribute-identities. It would lead too far to discuss all these objections. I confine myself to the remark that I am never impressed by objections against the distinctions between identities and correlations and between thing- and attribute-predicates of the type that there are examples which are difficult to classify. Moreover, it often occurs that explication of intuitively appealing distinctions takes a lot of formal logical work, of which the successful completion is anticipated by others, taking the risk for granted. A nice example is the distinction between ordinal and cardinal numbers.

In Section 2 the hypothesis of successful explicability of the distinction between correlations and identities was used to disentangle the explanation of laws in general and their reduction in particular. In Section 4 the same working hypothesis is presupposed in the analysis of concept reduction: identities will play a crucial role. Moreover, by way of precaution I will start with thing-predicates. In the second instance, however, it will turn out that the reduction of attribute-predicates runs completely analogous. By consequence, in any case this analysis does not provide an argument against attribute-identities as opposed to thing-identities.

4. Reduction of concepts

Reduction of concepts is of course conceived as the reduction, in different senses to be specified, of elementary statements in which the concept occurs. All concepts to be considered will be predicative. Hence, I will casually talk about reduction of predicates, in particular of thing- and attribute-predicates.

In Subsection 4.1. I will first decompose the reduction of the thing-predicate 'water' starting from a micro-token representation, into three steps: macro-classification, (existential) generalization and identification. Subsequently I will show how reduction with a detour via *micro-types*, if they can be defined independently of macro-types, is possible. This requires the steps micro-classification, generalization, macro-conversion and identification. Several notions can be defined in line with this analysis, e.g. the distinction between type-token and type-type reduction.

In Subsection 4.2. I will argue, using the temperature function as the leading example, that the reduction of attribute-predicates (representing properties, relations and functions) can take place essentially along the same lines, except that a time parameter is an unavoidable new element. I conclude this section with a formal survey (Subsection 4.3.).

In order to prevent unnecessary misunderstanding I like to remark that, in contrast to reduction of laws, reduction of concepts cannot be seen as a form of explanation. Hence, there is no explanation claim involved in this section.

The central claim of this section is to give an adequate analysis of different forms of reduction of concepts on the basis of some examples from elementary physics. Of course I hope to get in this way an instrument with which, together with other instruments such as the model for reduction of laws of Section 2, it is possible to deal with more complex but at the same time also more interesting reduction questions. The main challenge in this respect seems to be the mind-body problem, and a primary use-value (in sense (d)) may be to suggest new research for a materialistic account of the mind-body relation.

4.1. Reduction of thing-predicates

I shall take 'water' as an example of a thing-predicate. Consider the statement that a certain isolated amount of substance x_0 is water. It is plausible to call the level A of this 'object', and of this statement, the *macro-level* and the level B of molecules the *micro-level*. The elementary statement just mentioned thus gives a partial description or representation of a macro-object in macro-terms, for short called a partial *macro-*

representation. It is a *partial* macro-representation because for instance volume, pressure and temperature would also belong to a more or less complete macro-representation. In other words, x_0 is assigned one *macro-type* out of a coherent set of macro-types, namely mass-terms: it is a macro-type description or representation of x_0.

Let X indicate the set of macro-objects and let A more specifically refer to the relevant family of coherent macro-types, i.e. mass-terms (including terms for mixtures), with W in A for water. The macro-type representation function R specifies the macro-type of all x in X. Hence the elementary statement to be reduced can be written as: $R(x_0) = W$.

It should be stressed that the example has in the present context not yet anything to do with the aggregation phase of the substance water. That is, x_0 may as well be water in solid form (ice) or in gaseous form (vapour). The three phases are typical attributes (in particular properties) of the substance water.

Reduction of the statement "$R(x_0) = W$" of course means that we want to derive this statement from a suitable micro-representation of x_0 in terms of its molecules and (some of) their properties. Let B more specifically indicate the set of all conceptually possible *micro-structures* or *micro-tokens* of the following kind: a base set of (representations of) molecules, and for each molecule its chemical kind, e.g. H_2O. Let $s(x)$ in B, indicate the micro-structure description or *macro-token description* of x in X. By consequence, our task is to derive the statement "$R(X_0) = W$" from a statement of the type "$s(x_0) = b_0$".

I shall first disentangle this derivation in a small number of steps. In the second instance an instructive detour will be made.

I start with introducing some auxiliary means. The *projection* function Γ assigns to each macro-type a subclass of micro-tokens. For example, $\Gamma(W)$ is the set of micro-structures representing sets of H_2O-molecules. Note that this description alludes already informally to the relevant identity hypothesis, but formally the projection function does not invoke identities. I shall call $\Gamma(a)$, for a in A, the *projection class* of a and make the crucial assumption that the projection classes constitute together a partition of B: they don't overlap and their union is exhaustive. The latter is of course related to the assumption that every macro-object is assigned precisely one macro-type out of A, i.e. R is a function from X to A.

The second ingredient is the general *identity hypothesis* IH, stating that the statement that a certain macro-type applies is identical to the statement that a, further unspecified, micro-token applies that belongs to the projection class of that macro-type. Formally: $R(x) = a \equiv s(x) \in$ $\in \Gamma(a)$, where I use '≡' as identity-symbol between statements. The reader is referred to Section 3 for the distinction between identities and correlations.

The steps that are minimally needed to derive "$R(x_0) = W$" from "$s(x_0) = b_0$" are now the following. In the first step it is said to which projection class the given micro-token b_0 belongs, and I assume of course that this is indeed $\Gamma(W)$. This step will be called *macro-classification* of the micro-token. The next step abstracts from the specific micro-token and hence concludes that the unspecified micro-token representation $s(x_0)$ belongs to $\Gamma(W)$. As a matter of fact, this step is an application of the well-known, and undisputed, logical rule of inference called existential generalization (EG). For short, I shall just speak of *generalization*. From "$s(x_0) = b_0$" and "$b_0 \in \Gamma(W)$" the EG-step leads to the conclusion that "there is a b such that $s(x_0) = b$ and $b \in$ $\in \Gamma(W)$", which can of course be abbreviated as "$s(x_0) \in \Gamma(W)$" Finally, there comes the *identification* step: with the aid of the identity hypothesis IH it is derived that $R(x_0) = W$.

For obvious reasons the described reduction will be called (direct) macro-type-micro-token reduction or *heterogeneous type-token reduction* and can be summarized in the following scheme of steps.

	$s(x_0) = b_0$	micro-*token* representation
macro-classification	$b_0 \in \Gamma(W)$	
generalization (EG)	$s(x_0) \in \Gamma(W)$	
identification (IH)	$R(x_0) = W$	macro-*type* representation

The scheme shows clearly that type-token reduction consists of the derivation of a (macro-) type statement from a micro-token statement and hence the term "type-token reduction" should not be interpreted as a kind of equalization of a type-concept with a token-concept. Equalization only applies in the case of type-*type* reduction, to be dealt with later on.

It is important to stress that the foregoing analysis does no essential appeal to the existence of micro-types, that is, sets of micro-tokens which constitute, apart from macro-considerations, a natural whole and, hence can be defined independent of macro-types. In other words, (direct) reduction of concepts is possible without intervening micro-types, or more cautiously, at least reduction of thing-predicates is possible without assuming "natural kinds on the micro-level". Further on we will see that also reduction of *attribute*-predicates is possible without appeal to "natural attributes on the macro-level".

The foregoing does, however, not alter the fact that reduction often takes place with a detour via micro-types. This route will be described now and will turn out to be very illuminating.

The projection class $\Gamma(a)$ of the macro-type a is explicitly coupled to a, for which reason it cannot be said to be an autonomous micro-type. It seems plausible to define genuine *micro-types* in the present context as those equivalence classes of tokens of which the base sets all represent the same kind(s) of molecules. Although each micro-type of this sort will be assumed to coincide with the projection class of a certain macro-type, for the following it is of crucial importance that the micro-types can be defined in micro-terms independent of macro-types.

Using micro-types, the water example can now be analyzed in a still more sophisticated way as a concatenation of homogeneous type-token reduction and heterogeneous type-type reduction. Starting from the above definition of micro-types the *micro-classification function* ρ is introduced, specifying of each micro-token its respective micro-type in a non-trivial way: the kind(s) of molecules should explicitly be mentioned. Note that the micro-types constitute a partition of B and that $b \in \rho(b)$ is automatically fulfilled. On the basis of ρ the micro-type description or *micro-type representation function* r is defined as follows: $r(x) \underset{\mathrm{df}}{=} \rho(b)$ iff $s(x) = b$.

The first stage of the reduction-with-detour is called micro-type-micro-token reduction or *homogeneous type-token reduction* and comes schematically down to the following:

	$s(x_0) = b_0$	micro-*token* representation
micro-classification	$\rho(b_0) = K_0$	
generalization (EG)	$r(x_0) = K_0$	micro-*type* representation

In the first step the micro-type of b_0 is established, i.e. K_0, in the present case the class of sets of H_20-molecules. The second step abstracts from b_0.

For the second, and last, stage a modified version of the identity hypothesis is required. IH was defined in terms of the unspecified token $s(x)$ and may hence be called the *token-version*. The *type-version* $\mathrm{IH}^{=}$ is defined as: $R(x) = a \equiv r(x) = \Gamma(a)$.

The subsequent scheme of the remaining macro-type-micro-type reduction or heterogeneous type-type-reduction or, simply, *type-type reduction* now looks as follows:

	$r(x_0) = K_0$	micro-*type* representation
macro-conversion	$r(x_0) = \Gamma(W)$	
identification ($IH^=$)	$R(x_0) = W$	macro-*type* representation

In the first (or, in total, third) step it is established that the projection class of a certain macro-type coincides with, and hence may be called the *converse-side* of, the given micro-type. In the second (or fourth) step this coincidence is identified, on the basis of $IH^=$, with the intended macro-type representation.

It is easy to see that each example of *indirect* heterogeneous type-token reduction, that is an appropriate concatenation of homogeneous type-token reduction and (heterogeneous) type-type reduction, can be short-circuited to an example of *direct* heterogeneous type-token reduction. The type-version of the identity hypothesis can easily be transformed into the corresponding token-version, such that the micro-types can be left out, as is required for direct reduction.

It is a short-circuit, but the reverse is not always possible. It is by definition only possible to take the route-with-detour, instead of the short route, if it is possible to define exclusively on the basis of micro-means, micro-types that coincide with the projection class division of the micro-tokens. In the case of such 'natural' micro-types one frequently speaks of 'aggregate(d) concepts'. It deserves notice that these concepts do not presuppose aggregation in the sense of the so-called aggregation step in the micro-reduction of laws (whereas, conversely, such a step presupposes such concepts, *viz.* the terms in the resulting aggregated law).

Instead of a perfect detour there may also be an imperfect detour, which is nevertheless practicable, namely when the projection classes of the macro-types *approximately* coincide with the micro-types. Up to now all discussed forms of concept reduction were of *deductive* nature. Here we come upon *approximative* forms of reduction, in particular approximative heterogeneous type-type reduction (and hence also, in a derived sense, approximative indirect type-token reduction). *Macro-conversion* now means that the micro-type coincides *approximately* with the projection class of a certain macro-type. The identity hypothesis subsequently states that this is (necessary and) sufficient for the respective macro-statement.

For clarity's sake the foregoing exposition was presented in terms of type-token and type-type reduction, but it is of course also plausible to interpret the distinguished forms of type-token reduction as explication of the idea of *micro-reduction* of (thing-) concepts. Type-type reduction can then be called *iso-reduction*. As a matter of fact, the generalization step seems crucial for the type-token character of micro-reduction: in that step the specific token is released. Of course, whether or not there an identification step is occurring is decisive for the question whether it is heterogeneous or homogeneous reduction.

4.2. Reduction of attribute-predicates

"Water" is a typical thing-predicate, more in particular a mass-term. Assuming that the foregoing subsection gives an adequate analysis of the reduction of thing-predicates, the question is whether the analysis is also applicable to the reduction of attribute-predicates. There are three types of attributes: properties, relations and functions. In view of the fact that properties just as thing-predicates are indicated with unary predicates, it will be no surprise that the reduction of property-predicates has precisely the same form as the reduction of thing-predicates. The water example could have been analyzed as a combination of a thing-with-property: water in liquid form. It is indeed not difficult to see that the statement "x_0 is fluid" and the statement "x_0 is water in one of the three phases" can be reduced in a similar way. In the new example "x_0 is liquid" the relevant micro-structures should not have to specify the kind(s) of molecules involved, but the microscopic conditions for the liquid phase as well as for the gaseous and solid phases of a substance. The only thing which becomes immediately clear is that now it is advisable to introduce time as a variable, because a substance may undergo phase-transitions.

Let us now direct the attention to the reduction of a (quantitative) functional attribute-term: temperature. Consider the statement that the (absolute empirical) temperature θ of an isolated amount of gas (e.g. water-vapour!) at time t_0 is equal to T_0: $\theta(x_0, t_0) = T_0$. The macro-types are now of course the possible temperature values, i.e. the temperature scale, and θ is the macro-type representation function.

Also here the required micro-types deviate a bit from those of the preceding section. Now they consist of sets of molecules with specified mass, position and velocity. However, the statement from which the reduction has to start, is apart from the time parameter again of the same type: the micro-token representation $s(x_0, t_0) = b_0$.

As is well-known in the present case the informal derivation succeeds via the connecting concept of "mean kinetic energy" $\overline{u}(b)$, which is explicitly defined for any micro-token b in B. Stated in a compact way, one gets, according to the familiar functional relation $\overline{u} = (3/2)kT$ (where k is Boltzmann's constant), that "$\theta(x, t) = T$" is identical to the statement that the prevailing micro-state $s(x, t) = b$ is such that $\overline{u}(b) = (3/2)kT$ and hence that "$\theta(x_0, t_0) = T_0$" can be derived from "$s(x_0, t_0) = b_0$" if and only if $\overline{u}(b_0) = (3/2)\ kT_0$.

For the spelled out argument the projection function is of course defined as follows: $\Gamma(T) =_{df} \{b \in B/\overline{u}(b) = (3/2)kT\}$, and the identity hypothesis states: $\theta(x, t) = T \equiv s(x, t) \in \Gamma(T)$. The direct heterogeneous micro-reduction now proceeds as follows: micro-token representation: $s(x_0, t_0) = b_0$, macro-classification: $b_0 \in \Gamma(T_0)$, generalization: $s(x_0, t_0) \in \Gamma(T_0)$, and finally identification on the basis of IH: $\theta(x_0, t_0) = T_0$.

In this example of attribute reduction, like in the foregoing, the detour via micro-types is possible. It suffices here to state the relevant definition of a micro-type: $K(u) = \{b \in B/\overline{u}(b) = u\}$. Notice that here, even more clearly than in the water example, it is possible to indicate the distinction between the definition of a projection class and of a micro-type, because in the first one temperature is explicitly mentioned in the definiens, whereas in the second it is not. In the water example the corresponding explicit mentioning is absent, making the fundamental difference between projection classes and micro-types in that case invisible.

As mentioned in Section 3, the present identity between two quantitative concepts can be obtained from the identification "equal thermal state is the same as equal mean kinetic energy", where the macro-attribute ("equal thermal state") is a qualitative (equivalence) *relation*. Now it is not difficult to check that the above sketched reduction can be transformed into the reduction of this relation, by which we have indicated an example of reduction of the third category of attributes, beside properties and functions.

By this it has sufficiently been argued that the reduction of attribute-predicates can take place in a completely analogous way to the reduction of thing-predicates. Only the introduction of the time parameter seems now unavoidable.

Even for somebody who rejects not only attribute-identities but also thing-identities the foregoing analysis is not yet entirely useless, for it is not difficult to transform the account into a model for concept *correlation*. Consider for this purpose instead of the (token-version of the) identity hypothesis the *correlation hypothesis*, stating that there is a *causal* relation between macro-type a and its projection class $\Gamma(a)$ such that, just as in the case of the identity hypothesis, the biconditional "$R(x, t) = a$ iff $s(x, t) \in \Gamma(a)$" follows. Replacement of the identification step in the scheme for (direct) heterogeneous micro-reduction by a *correlation step*, based on the mentioned correlation hypothesis, leads now to the scheme for (direct) heterogeneous *micro-correlation*. Heterogeneous *iso-correlation* can be defined in an analogous way. The two main questions of Section 3 can now be summarized by: is there only correlation of concepts or is reduction (by identification) also possible, and if so, is there only reduction of thing-concepts or also of attribute-concepts?

4.3. *Formal survey*

By way of summary this subsection gives a survey of the formal aspects of the foregoing two subsections, leaving out all example related notation, and including the time parameter throughout.

Sets and functions

- X set of macro-objects
- T time axis
- A family set of macro-types
- R $X \times T \to A$, macro-type representation function (macro-type description)
- B set of conceptually possible micro-structures/micro-tokens
- s $X \times T \to B$, micro-token representation function (micro-structure description)
- Γ $A \to P(B)$, projection function, leading to the family of projection classes $\Gamma(a)$, $a \in A$, which constitutes a partition of B ($P(B)$: the power set of B)
- ρ $B \to P(B)$, micro-classification function, leading to the family of micro-types $\rho(b)$, $b \in B$, which constitutes a partition of B

r $X \times T \rightarrow P(B)$, micro-type representation function (micro-type description), defined by: $r(x, t) \underset{\text{df}}{=} \rho(b)$ iff $s(x, t) = b$

$\approx$ $r(x, t) \approx \Gamma(a)$: $r(x, t)$ approximately coincides with $\Gamma(a)$.

Identity hypotheses

IH token-version:	$R(x, t) = a \equiv s(x, t) \in \Gamma(a)$
$\text{IH}^{=}$ type-version:	$R(x, t) = a \equiv r(x, t) = \Gamma(a)$
$\text{IH}^{\approx}$ approximative type-version	$R(x, t) = a \equiv r(x, t) \approx \Gamma(a)$

Basic types of concept reduction

heterogeneous (type-token) micro-reduction (direct)

	$s(x_0, t_0) = b_o$ micro-token representation
macro-classification	$b_0 \in \Gamma(a_0)$
generalization (EG)	$s(x_0, t_0) \in \Gamma(a_0)$
identification (IH)	$R(x_0, t_0) = a_o$ macro-type representation

homogeneous (type-token) micro-reduction

	$s(x_0, t_0) = b_0$ micro-token representation
micro-classification	$\rho(b_0) = K_0$
generalization (EG)	$r(x_0, t_0) = K_0$ micro-type representation

heterogeneous (type-type) iso-reduction: deductive/approximative

	$r(x_0, t_0) \in K_0$ micro-type representation
macro-conversion	$r(x_0, t_0) =/\approx \Gamma(a_0)$
identification ($\text{IH}^{=}/\text{IH}^{\approx}$)	$R(x_0, t_0) = a_0$ macro-type representation

indirect heterogeneous (type-token) micro-reduction

homogeneous micro-reduction, followed by
heterogeneous iso-reduction.

In our treatment of concept reduction, the point of departure was always a macro-*type* representation. Of course, a macro-*token* representation is the combination of a number of macro-type representations, such that each macro-type belongs to a different family of macro-types, while the relevant set of families is considered as (relatively) complete. By consequence, the presented analysis can directly be used to describe the reduction of macro-token representations.

Finally, in the next section I will make explicit an underlying ontological assumption of the treated examples of (micro-) reduction of concepts.

5. *Levels of reduction*

In discussions about reduction it is almost inevitable to talk about levels. However, at the same time, it is usually done in a rather confusing way, not realizing enough that the situation is indeed very complicated. In this section I like to streamline the talk about levels of reduction.

Both examples of concept reduction are clear cases in which two related ontological levels play a role, namely the level of macro-objects and that of micro-objects, i.e. molecules. This distinction of ontological levels is of course based on the fact that the micro-objects are in the most literal sense parts of the macro-objects. Now it is important to note that a micro-token is not related to *one* micro-object, i.e. one molecule, but to all the molecules composing the relevant macro-object. hence, between a micro-token at the one hand and a micro-type or (the projection class of a) macro-type at the other there is no ontological part-whole relationship but an abstract element-set relationship (on which an existential generalization step can be based). Consequently, micro-reduction of concepts as defined thusfar does not yet presuppose part-whole related ontological levels. It seems nevertheless advisable to speak only of micro-reduction of concepts if not only existential generalization (from token to type), but also two part-whole related ontological levels are involved. I will call the latter assumption the ontological presupposition. In the case of micro-reduction of laws this extra condition is (only) redundant, because the crucial aggregation step guarantees almost by definition the presence of two part-whole related ontological levels. It is tempting to formulate now, in passing, the most plausible explication of the idea of *ontological reduction*: micro-reduction of thing-concepts. Although this explication deviates formally quite a lot from the one by Moulines [1984] it is close to it in spirit.

It is less easy to distinguish, in the line of the foregoing, conceptual or epistemological levels. The following four kinds of concepts are in any case involved in this question: macro-types, projection classes of macro-types, micro-types and micro-tokens; they are members of A, $P(B)$, $P(B)$ and B, respectively. It seems plausible to speak also of an epistemological macro- and micro-level, and to situate on these levels in any case macro-types and micro-tokens, respectively. A problem which remains is the localization of projection classes and micro-types. A relatively easy solution is the following. The order of magnitude of the number of distinguished micro-types (of one family) will in general be the same as that of the number of distinguished macro-types (of a more or less corresponding family) and, hence, also as that of the number of projection classes. In view of the further nature of projection classes and micro-types they can be considered as different epistemological sides of the (ontological cum epistemological) macro-level of macro-objects and macro-types. The micro-types constitute as it were the converse side of the projection classes of the macro-types, and may coincide strictly or approximately. Both are ways of division, more in particular partition, of the set of micro-tokens, the one attached to macro-types, the other without appeal to macro-means but only to micro-means.

In sum, to the ontological macro-level corresponds an epistemological macro-level, which has an upper and a lower side in terms of sets of micro-tokens. The micro-tokens themselves constitute the epistemological micro-level, which is directly related to the ontological micro-level.

6. Concluding considerations

The first question to be discussed in this final section is the relation between the two kinds of reduction. To begin with, it is clear that none of the three variants of concept reduction presupposes (some form of) reduction of laws. Hence, the question is whether, conversely, reduction of laws presupposes concept reduction? Let us disentangle this question by considering the five steps of the decomposition model of explanation of a law. It is again clear that the two steps application and correlation, which on their own were claimed not to justify the speaking of reduction of the law, have nothing intrinsically to do with concept reduction. Consider next the approximation step, the occurrence of which was one of the reasons in the literature to speak of reduction of the law in question. It has already been mentioned that this step can be seen as the result of reduction in the strategic sense, i.e. idealization and subsequent concretization. Hence, although this step does not involve concept reduction in one of the transformation senses, it does involve concept reduction in an, as yet, informal strategic sense: some aspects, and hence the concepts representing them, which were first neglected, have later been accounted for; going backwards, they may again be deleted in the approximation step.

It is evident that the two remaining steps, aggregation and identification, have something to do with concept reduction in the transformation sense. Note first that the identification step presupposes by definition one or more identity hypotheses. Now it seems that, if an aggregation step occurs as well, the identity hypotheses are more specifically of the type-version, as we have met them in the context of concept reduction. Hence, the following claim seems plausible: if an aggregation step is followed by an identification step, the latter consists of the carrying out of all relevant (heterogeneous) type-type reductions of concepts, more precisely, all "aggregate concepts" i.e. micro-type concepts, occurring in the aggregated law are replaced by their macro-conversion-counterpart, while the relevant identity hypotheses guarantee intrinsic preservation of truth value during this transformation.

Now what if there is identification but no aggregation? At the end of Subsection 2.3. I left it as a challenge to find proper examples of the corresponding pure type of reduction of a law. If it exists, then, again, it seems plausible that type-type concept reduction is involved. One particular possible form shall be mentioned here. There may be a macro-law for which we can find and establish a micro-version, that is a law formulated in terms of micro-types, which can be transformed into the original macro-law by the relevant type-type concept reductions. Because micro-types are also called aggregate(d) concepts, the discovered micro-law might be called an *aggregate law*. But this does not imply that it is an *aggregated law*, in the sense of Section 2, i.e. the aggregation of the effect of an individual law applied to a collection of objects. The aggregate law is only then also an aggregated law if it turns out to be possible to micro-reduce it by such an aggregation step. Hence, we may look in particular for a macro-law which can be deductively explained by an aggregate law (solely) on the basis of identification, whereas it is not yet known whether the aggregate law itself can be micro-reduced. If the indicated explanation is called reduction, we have a pure example of identificatory reduction, although we should not exclude that calling it reduction may still be an implicit anticipation of successful micro-reduction.

In both cases discussed thusfar, that is, identification with or without aggregation, there is of course also room for approximative variants, i.e. quasi-identification steps based on approximative (type-versions of) identity hypotheses.

Let us finally consider the aggregation step. At first sight it may seem as if an aggregation step comes down to systematically carrying out all relevant homogeneous type-token concept reductions. But on closer inspection this is not true at all, I have already warned against the misinterpretation that the notion of "aggregate concept" would presuppose an aggregation step in the present sense.

Here, I should add the warning that the (existential) generalization step characteristic for type-token concept reduction should not be misinterpreted as the aggregation step which is characteristic for micro-reduction of a law: the two steps do in fact quite different jobs. In sum, there is no good reason to relate homogeneous micro-reduction of a law to homogeneous micro-type-token) reduction of concepts. By consequence, if an aggregation step is followed by an identification step, their combination is not related to *indirect* heterogeneous type-token reduction of concepts, although, as we have seen, the identification step is related to heterogeneous type-type reduction of concepts.

The (mistaken) idea that the aggregation step in itself involves some kind of concept reduction may well be due to the fact that aggregation presupposes the definability of "aggregate (d)concepts", i.e. of micro-types. This implies on its turn the very possibility of homogeneous type-token concept reduction. But this does not imply that this possibility is realized in the aggregation step.

The second question to be discussed is the relation between the presented epistemological analysis of reduction of laws and concepts and all kinds of metaphysical positions and methodological research strategies. This final part is an attempt to indicate the different use-values of the presented models (in particular (b), (d), and (e), see Section 1). In the discussion I will restrict myself to *vertical reduction*, that is, reduction of concepts in one of the explicated senses, and reduction of laws based on aggregation and/or identification. In all these cases two (ontological and/or epistemological) levels are involved. This leaves reduction of laws based on approximation apart as a kind of *horizontal reduction*. From here on "reduction" is to be read as "vertical reduction", except when otherwise stated.

Let me start with formulating three philosophical (ontological cum epistemological) positions with respect to a certain "macro-"domain. They are formulated as general statements about the possibility of reduction of the concepts and laws of that domain. *Radical* (philosophical) *reductionism* is the belief that every macro-concept and every macro-law can be reduced. *Radical holism* is the belief that all concepts and laws of the macro-domain cannot be reduced. Finally, *restricted reductionism* (*and holism*!) is the belief that some concepts and laws will be reducible, but others not.

As a side-remark I like to draw attention to the intriguing question whether a law may not be reducible but nevertheless explainable by some micro-theory due to the fact that the required transformation rules are irreducible correlations, constituting a complex causal bridge theory [Maull, 1977].

It is plausible to formulate also three methodological approaches or research strategies with respect to a certain macro-domain. In the radical *reductionistic strategy* all attention is directed to the reduction of macro-laws and -concepts and the establishment of aggregate concepts and aggregated laws. In the *radical holistic strategy* all attention is directed to formation of macro-concepts, to discovering macro-laws, and finally explanation of macro-laws by encompassing macro-theories (hence, without using aggregation or identification). Finally, there is the *mixed strategy*, according to which all these activities take place alternately, depending on all kinds of considerations, such as the stage of the research etc.

It is important to note that the terminologically corresponding strategies and positions are not strictly coupled, except perhaps that radical philosophical holism leaves only room for the radical holistic strategy. The converse is not self-evident. There are excellent examples of research according to the radical holistic strategy, e.g. phenomenological thermodynamics and macro-economics, where it is seen as a compatible, but separate task to try to reduce the macro-concepts and -laws, i.e. to work according to the radical reductionistic strategy. To be sure, it has to be conceded that reductionistic strategies in general have been very successful in the history of science.

However, the radical reductionistic strategy leads many times to impressive minute research, constituting perfect aggregate concepts and aggegated laws, where, however, nobody is waiting for. On the other hand, the radical holistic strategy frequently degenerates into hardly testable and transferable insights.

In line with these roughly formulated impressions, it is plausible to formulate the working hypothesis that the mixed strategy will be in many cases the best strategy. For, to begin with, to reduce concepts and laws of a certain domain, they have first to be established. On its turn, it is frequently the case that the search for concepts and laws has been stimulated very much by reductionistic questions.

Research according to the mixed strategy I here conceive more specifically as an important case of interdisciplinary research in the sense of Henk Zandvoort's explication [1986]. According to Zandvoort (successful) *interdisciplinary research* consists of the cooperation of two or more research programs, belonging to one or more disciplines, where the cooperation takes place according to the following rules of the game:

— one program is the *guide-program*, which raises problems of theoretical (e.g. explanatory) or experimental nature to the others,

— the other programs are service- or *supply-programs*, which have successfully passed their test phase and hence can try to solve the problems provided by the guide-program.

Besides this asymmetric form there is a related symmetric form of interdisciplinary research consisting of, say, two programs which are both still in their testphase and which play alternately the role of guide- and supply-program.

In the case of *reductive* interdisciplinary research the guide program is of course a program on the macro-level, whereas a supply-program deals with the micro-level.

Henk Zandvoort's paradigmatic (asymmetric) example of a supply program in the natural sciences is the NMR-(nuclear magnetic resonance) program. An important (asymmetric) example in the social sciences is the utility maximization program. It has proved its strength in micro-economics and nowadays it cooperates with guide programs in macro-economics and macro-sociology.

An historical example of the symmetric (and reductive) type is the cooperation between phenomenological thermodynamics and statistical mechanics. An example to be dreamt of concerns the cooperation of psychology and (neuro-) physiology, i.e. the scientific research of the mind-body problem, also mentioned, at the beginning of Section 4, as a long term target of the analysis of concept reduction.

The indicated type of interdisciplinary research fits well with the following view on different types of explanation. Beside "vertical" forms of reductive explanation, there are not only other vertical forms of non-reductive explanation (e.g. based on correlation), but also many types of "horizontal" explanation. As far as nomological explanation of laws is concerned all these types of explanation seem to fit well in our general decomposition model. However, there have always been claims that there are types of explanation which are essentially different from nomological explanation. In Kuipers [1986] I have argued that e.g. intentional explanation and functional explanation (in biology) constitute indeed autonomous forms of explanation and that they fit into a general model of so-called *explanation by* (intentional, functional, etc.) *specification*, which also includes certain types of causal explanation. The relevance for the present context is the following: types of explanation by specification are important forms of horizontal explanation, and hence they may be the dominant way of explanation in a guide program. Consequently, instead of disqualifying explanation by specification it may play a crucial role in reductive interdisciplinary mind-body research: it can open the way for interesting forms of reductive physiological explanation, a way that would be indistinguishable in the overwhelming amount of possibilities confronting the radical reductionistic strategy. Conversely, reductive physiological explanation may very well suggest new ways for further development of psychological explanation, a possibility which is blocked by the radical holistic strategy.

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NOTES

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